Understanding Human Optimization: The Case for a Tractable-design Cycle

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Overview

Formulating cognitive theories
  Levels of Explanation
  Optimization and Satisficing

Testing cognitive theories
  Underdetermination of Theory
  Utilizing Theoretical Constraints

Tractable-design cycle
  Cautions and Clarifications

Conclusion
Levels of Description

input $i$ → Cognitive process → output $o = f(i)$

<table>
<thead>
<tr>
<th>Level</th>
<th>Marr’s levels</th>
<th>Question</th>
</tr>
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<tbody>
<tr>
<td>1</td>
<td>Computational</td>
<td>What?</td>
</tr>
<tr>
<td>2</td>
<td>Algorithm</td>
<td>$How_1?$</td>
</tr>
<tr>
<td>3</td>
<td>Implementation</td>
<td>$How_2?$</td>
</tr>
</tbody>
</table>
Underdetermination of Lower Levels

Computational level

Algorithmic level

Implementational level

\[ \begin{array}{c}
F_1 & F_2 & F_3 & \ldots \\
A_{21} & A_{22} & A_{23} & A_{24} & \ldots \\
M_{231} & M_{232} & M_{233} & M_{234} & \ldots \\
\end{array} \]
<table>
<thead>
<tr>
<th>Domain</th>
<th>Computational Level Theory (Informal)</th>
<th>References</th>
</tr>
</thead>
<tbody>
<tr>
<td>Similarity</td>
<td>Input: Two objects, x and y. Output: The length of the shortest program computing x from y.</td>
<td>(Hahn, Chater, &amp; Richardson, 2003; Chater and Vitanyi, 2003)</td>
</tr>
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</table>
Categorization (informal)

Input: A set of objects.

Output: A partition that maximizes within-category similarity and between-category dissimilarity.

Categorization (formal)

Input: A set of objects, \( A \), with a similarity measure \( s(x,y) \) and a dissimilarity measure \( d(x,y) \) for each pair of objects \( x, y \in A \).

Output: A partition of \( A \) into categories \( A_1, A_2, \ldots, A_k \), such that

\[
\sum_{x,y \in A_i, i=1,2,...,k} s(x,y) + \sum_{x,y \notin A_i, i=1,2,...,k} d(x,y)
\]

is maximum.
Formalization Example 2

**Coherence** (informal)

*Input:* A set of interconnected beliefs.

*Output:* A truth assignment of maximum coherence.

**Coherence** (formal)

*Input:* Set of propositions $P$, set of constraints $C = C^- \cup C^+$.  

*Output:* A truth assignment to the propositions in $P$ that satisfies a maximum number of constraints.

A constraint $(p, q) \in C^-$ is satisfied if $p$ is ‘false’ and $q$ is ‘true’.

A constraint $(p, q) \in C^+$ is satisfied if both $p$ and $q$ are ‘true’ or both $p$ and $q$ are ‘false’.
Empirical Underdetermination of the Computational Level

Empirical data

- Empirical data
  - D₁, D₂, D₃, D₄, D₅, D₆, ...

Computational level

- Computational level
  - F₁, F₂, F₃, ...

Algorithmic level

- Algorithmic level
  - A₂₁, A₂₂, A₂₃, A₂₄, ...

Implementational level

- Implementational level
  - M₂₃₁, M₂₃₂, M₂₃₃, M₂₃₄, ...

Consistent
Empirical Underdetermination of the Computational Level

Several reasons

1. Any finite set of input-output observations is consistent with infinitely many different functions.
2. Inputs and outputs are usually not directly observable.
3. Psychological data are noisy (due to context variables not under the control of the experimenter).
4. Commitment is usually to the informal theory, not the formalization.
Even More Underdetermination …

D_1 \quad D_2 \quad D_3 \quad D_4 \quad D_5 \quad D_6 \quad D_7 \quad …

\[ \text{inconsistent} \]

F_1 \quad F_2 \quad F_3 \quad …

\[ \text{consistent} \]

A_{21} \quad A_{22} \quad A_{23} \quad A_{24} \quad …

\[ \text{consistent} \]

M_{231} \quad M_{232} \quad M_{233} \quad M_{234} \quad …

\[ \text{consistent} \]

I_1 \quad I_2 \quad I_3 \quad …

\[ \text{consistent} \]
Can We Use Lower-Level Constraints?

$D_1 \rightarrow D_2 \rightarrow D_3 \rightarrow D_4 \rightarrow D_5 \rightarrow D_6 \ldots$

$F_1 \rightarrow F_2 \rightarrow F_3 \ldots$

$I_1 \rightarrow I_2 \rightarrow I_3 \ldots$

$\text{consistent}$
Can We Use Lower-Level Constraints?

D_1, D_2, D_3, D_4, D_5, D_6, ...

consistent

F_1, F_2, F_3, ...

I_1, I_2, I_3, ...

consistent

Computability

Tractability
Computability Constraint

Cognitive functions $\subseteq$ Computable functions
Tractability Constraint

Observation 1:
Cognitive functions are implemented by physical systems.

Observation 2:
Physical systems are limited in space and speed.

Conclusion:
Cognitive functions $\subseteq$ Tractable functions.

[e.g. Frixione, 2001; Simon, 1990; Thagard & Verbeurgt, 1998]
Tractability Constraint

Cognitive functions $\subseteq$ Tractable functions
Is Rosch’s Categorization Tractable?

**Categorization** (informal)

*Input:* A set of objects.

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**Categorization** (formal)

*Input:* A set of objects, $A$, with a similarity measure $s(x,y)$ and a dissimilarity measure $d(x,y)$ for each pair of objects $x, y \in A$.

*Output:* A partition of $A$ into categories $A_1, A_2, \ldots, A_k$, such that

$$\sum_{x,y \in A_i, i=1,2,\ldots,k} s(x,y) + \sum_{x,y \not\in A_1, i=1,2,\ldots,k} d(x,y)$$

is maximum.
Is Thagard’s Coherence Tractable?

**Coherence** (informal)

*Input*: A set of interconnected beliefs.

*Output*: A truth assignment of maximum coherence.

**Coherence** (formal)

*Input*: Set of propositions $P$, set of constraints $C = C^- \cup C^+$.

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Unbounded Exponential-time Computation is Intractable

Exhaustive search of combinatorial complex spaces is impractical for all but very small input sizes.

<table>
<thead>
<tr>
<th>$n$</th>
<th>$O(n^2)$</th>
<th>$O(2^n)$</th>
</tr>
</thead>
<tbody>
<tr>
<td>5</td>
<td>0.15 msec</td>
<td>0.19 msec</td>
</tr>
<tr>
<td>20</td>
<td>0.04 sec</td>
<td>1.75 min</td>
</tr>
<tr>
<td>50</td>
<td>0.25 sec</td>
<td>8.4 x $10^2$ yrs</td>
</tr>
<tr>
<td>100</td>
<td>1.00 sec</td>
<td>9.4 x $10^{17}$ yrs</td>
</tr>
<tr>
<td>1000</td>
<td>1.67 min</td>
<td>7.9 x $10^{288}$ yrs</td>
</tr>
</tbody>
</table>
Rosch’s Categorization and Thagard’s Coherence are NP-hard

Cognitive functions $\subseteq$ Tractable functions
Empirical Cycle + Tractable-design Cycle

Computational level theory
  - Informal
  - Formal

Empirical testing

Tractability testing
Cautions and Clarifications

Intractability is not always bad news!
(Or, at least: don’t shoot the messenger)
Cautions and Clarifications

Tractability is not always good news!
(Or, at least: it is not a goal in itself)
Cautions and Clarifications

Tractability is not trivially achieved!

For example:
Optimization is tractable $\iff$ Satisficing is tractable

**Coherence** (optimization variant)

*Input:* Set of propositions $P$, set of constraints $C = C^- \cup C^+$.  
*Output:* A truth assignment to the propositions in $P$ that satisfies a **maximum** number of constraints.

**Coherence** (satisficing variant)

*Input:* Set of propositions $P$, set of constraints $C = C^- \cup C^+$, integer $k$.  
*Output:* A truth assignment to the propositions in $P$ that satisfies **at least** $k$ constraints.
Cautions and Clarifications

Heuristics cannot serve as algorithmic level theories!
Cautions and Clarifications

Intractability requires theory change!
Cautions and Clarifications

Domain restriction is a form of theory change!

For example:

**Coherence** (unrestricted)

*Input:* Set of propositions $P$, set of constraints $C = C^- \cup C^+$.

*Output:* A truth assignment to the propositions in $P$ that satisfies a maximum number of constraints.

**Coherence** (restricted)

*Input:* Set of propositions $P$, set of constraints $C = C^- \cup C^+$, *such that property X holds.*

*Output:* A truth assignment to the propositions in $P$ that satisfies a maximum number of constraints.
Cautions and Clarifications

Get the most out of tractability analysis!
For example: Analyse many (embedded) formalizations
Summary & Conclusion

Benefits of Tractable-Design Cycle
- Encourages formalization
- Helps constrain computational-level theory
- Understanding of cognitive (im)possibilities

Cautions and Clarifications

Open methodological question
- How to assess (in)tractability of theories?